perhaps induced by an electric field associated with a wave if the electrons were originally at rest, then the energy change W_{pe} would be positive.

therefore $\Delta W = W_{pe} = \frac{1}{2} m_e v_1^2$ if $v_0 = 0$

if the electron were originally drifting with velocity $\vec{v}_0 = v_0 \hat{x}$

Then the energy change due to the perturbation would be negative,

therefore,

$$\Delta W = W_{pe} = \frac{1}{2} m_e (v_0 - v_1)^2 - \frac{1}{2} m_e v_0^2$$

$$\simeq - m_e v_0 v_1$$

The negative energy comes from the fact that the electrons had a drift velocity v_0. The existence of negative energy waves in general implies that a drift or ~~being~~ beam in the plasma

Ion - Acoustic Wave :- (5.4)

Ion-acoustic wave or ion sound wave in plasma is a low frequency wave in which ions play a important roles. Here ions form regions of compressions and ~~larger~~ rarefaction just as in an ordinary sound wave in a gas. As it involves the motions of massive ions, it is a low frequency oscillations and both electrons and ions will have time to move. For analytical study here we use the neutrality condition of plasma approximations that is $n_i = n_e = n_0$ and do not use poisson's eqns for electrostatic oscillations the two fluid eqns are –

$$\frac{\partial n_j}{\partial t} + \vec{\nabla}\cdot(n_j \vec{u}_j) = 0 \quad \text{——} (1)$$

$$m_j n_j \left[\frac{\partial \vec{u}_j}{\partial t} + (\vec{u}_j\cdot\vec{\nabla})\vec{u}_j\right] = q_j n_j \vec{E} - \gamma_j k_B T_j \vec{\nabla} n_j \quad \text{——} (2)$$

~~When~~

where subscript $j = e$ for electrons and $j = i$ for ions.

Since the electrons can move much faster than the heavier ions and we consider low frequency oscillation so that electron inertia may be neglected.

Using $m_e = 0$ and $\vec{E} = -\vec{\nabla}\phi$, we get from (2)

$$e n_e \vec{\nabla}\phi - \gamma_e k_B T_e \vec{\nabla} n_e = 0 \quad \text{——} (3)$$

for slow ion waves the electrons move so fast that they have enough time to equalize their temperature everywhere. So the electrons may be considered isothermal. We can take $\gamma_e = 1$.

Therefore, eqn (3) may be written in one dimension as –

$$e\frac{d\phi}{dx} = \frac{k_B T_e}{n_e}\frac{dn_e}{dx} \quad \text{——} (4)$$

$$\Rightarrow e\,d\phi = k_B T_e \frac{dn_e}{n_e}$$

Integrating both sides we get

$$e\phi = k_B T_e \ln n_e + C$$

In equilibrium situation $n_e = n_0$ and $\phi = 0$

$$0 = k_B T_e \ln n_0 + C$$

$$C = -k_B T_e \ln n_0$$

Then we have

$$e\phi = k_B T_e \ln \frac{n_e}{n_0}$$

$$n_e = n_0 \exp\left[\frac{e\phi}{k_B T_e}\right] \quad \text{--- (5)}$$

Hence the electrons are Boltzmann distributed. We assume the perturbation to be small relative to thermal energy. That is $\phi << k_B T_e$. Therefore from (5) we get

$$n_e \approx n_0 \left[1 + \frac{e\phi}{k_B T_e}\right]$$

So the perturbation in electron density is

$$n_{e1} = n_e - n_0 = n_0 \frac{e\phi}{k_B T_e} = n_{i1} \quad \text{--- (6)}$$

We assume small amplitude waves so that perturbations in plasma parameters are small as compared to their equilibrium values. Let

$$\left.\begin{aligned} n_j &= n_0 + n_{j1} \\ \vec{U}_j &= \vec{U}_{j0} + \vec{U}_{j1} \\ \phi &= \phi_0 + \phi_1 \end{aligned}\right\} \text{--- (7)}$$

where the subscript 0 refers to equilibrium values and subscript 1 refers to the perturbed part. For a uniform neutral plasma at rest in the equilibrium state

$$\vec{U}_{j0} = \phi_0 = 0$$

$$\vec{\nabla} n_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial \vec{U}_{j0}}{\partial t} = \frac{\partial \phi_0}{\partial t}$$

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Substituting the perturbation expansion (7) in eqns (1) and (2) for ions using (8) we get —

$$\frac{\partial n_{i1}}{\partial t} + n_0 \vec{\nabla} \cdot \vec{u}_{i1} = 0 \qquad (9)$$

$$m_i n_0 \frac{\partial \vec{u}_{i1}}{\partial t} = - e n_0 \vec{\nabla} \phi_1 - \gamma_i k_B T_i \vec{\nabla} n_{i1} \qquad (10)$$

where we have assumed the changes n_{i1}, $\vec{u}_{i1}$, ϕ_1 due to perturbations to be small quantities of 1st. order & neglected higher order smaller term. ~~Now~~

Now considering one dimensional case and assuming that all the perturbation quantities vary as ~~exp~~ $e^{i(kx-\omega t)}$ we get from eqn (9) and (10)

$$-i\omega n_{i1} + n_0 i k u_{i1} = 0 \qquad (11)$$

$$-m_i n_0 i \omega u_{i1} = -i e n_0 k \phi_1 - i \gamma_i k_B T_i k n_{i1} \qquad (12)$$

eleminating u_{i1} from eqn (11) & (12) and using (6) we get.

$$u_{i1} = \frac{i\omega n_{i1}}{i n_0 k} = \frac{\omega n_{i1}}{n_0 k}$$

$$-i m_i n_0 \omega \left(\frac{\omega n_{i1}}{n_0 k}\right) = -i e n_0 k \frac{n_{i1} k_B T_e}{n_0 e} - i \gamma_i k_B T_i k n_{i1}$$

From (6): $\phi_1 = \frac{n_{i1} k_B T_e}{n_0 e}$

$$\Rightarrow -\frac{m_i \omega^2}{k} = -n_0 k k_B T_e - \gamma_i k_B T_i k$$

$$\Rightarrow \frac{\omega^2}{k^2} = \frac{k_B T_e + k_B T_i \gamma_i}{m_i}$$

$$\Rightarrow \frac{\omega}{k} = \left[\frac{k_B T_e + k_B T_i \gamma_i}{m_i}\right]^{\frac{1}{2}} \quad \text{—— (13)}$$

$$= v_s \quad \text{—— (14)}$$

(ion acoustic wave)

This is the dispersion relation for Ion-acoustic waves; where v_s is the ion acoustic speed or ion sounds speed in plasma. Since ions suffer one dimensional perturbation in the plane waves we may take $\gamma_i = 3$.

The dispersion relation (14) shows that under plasma approximation the ion-acoustic waves moves with constant phase velocity and it does not show any dispersion. Here group velocity is equal to phase velocity.

When $T_i \to 0$ ion acoustic waves can still propagate. It cannot happen for ordinary sound waves in a gass which propagates due to collisions. In plasma collisions are not important because here sound waves can propagate through the intermediary of an electric field. Ions transmit vibration to each other because of their charge. In the limit $T_i \to 0$ the ion sound wave propagates with velocity

$$V_s = \sqrt{\frac{K_B T_e}{m_i}} \quad (V_s \propto T_e)$$

It involves electron temperature T_e because electric field is proportional to it and ion mass because of fluid inertia is proportional to it also.

Note **Electro static electron oscillations in present of magnetic field:-**

When a magnetic field exist we shall examine only the simplest cases. To derive an expressions for dispersion relation of the electrostatic wave in presence of magnetic field. Let us assume the following assumptions

i) There is a static magnetic field B_0 in the direction $\perp$ to x-axis. That is along z-axis. Therefore $\vec{B} = B_0 \hat{z}$

ii) The ions are fixed in space in a uniform distribution.

iii) There are no thermal motion that is $KT_e = 0$

iv) The plasma is infinite in extended

v) The electron oscillation occure only the x-direction

(vi) The plasma is neutral at rest.

From the above set of assumptions we have the electrons eqn of motions are —

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}\cdot(n_e \vec{v_e}) = 0 \quad \text{——— (1)}$$

$$\frac{\partial \vec{v_e}}{\partial t} + (v_e \cdot \vec{\nabla})\vec{v_e} = -\frac{eE}{m_e} - \frac{e}{m_e c}(v_e \times B) \quad \text{——— (2)}$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi e(n_0 - n_e) \quad \text{——— (3)}$$

where $\vec{v_e}$, n_e and $\vec{E}$ are the electron velocity, electron number density and electric field and n_0 is the background ion density. | $\epsilon_0 = \frac{1}{4\pi}$

eqns (1), (2) & (3) can be easily solve by the procedure of linearization.

By this, we mean that the amplitude of oscillation is small and terms containing higher powers of amplitude factors can be neglected. We first separate the dependent variable in two parts: an equilibrium part indicated by subscript 0 and an perturbation part indicated by subscript 1

Thus we have

$$\left.\begin{aligned} \vec{B} &= \vec{B_0} + \vec{B_1} \\ \vec{E} &= \vec{E_1} \\ \vec{v_e} &= \vec{v_{e1}} \\ n_e &= n_0 + n_{e1} \end{aligned}\right\} \quad \text{——— (A)}$$

using A, then the eq^n (1), (2) & (3) can be written as $\frac{\partial n_{e1}}{\partial t} + n_0 \vec{\nabla}\cdot\vec{v}_{e1} = 0$ —— (4)

$$\frac{\partial \vec{v}_{e1}}{\partial t} = -\frac{eE_1}{m_e} - \frac{e}{m_e c}\left(\vec{v}_{e1}\times\vec{B}_0\right) \quad \text{—— (5)}$$

$$\vec{\nabla}\cdot\vec{E}_1 = -4\pi e\, n_{e1} \quad \text{—— (6)}$$

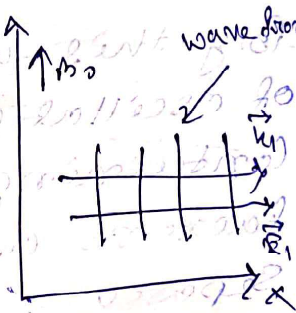
we shall consider only ~~longitudnal~~ longitudinal wave with $\vec{k} \parallel \vec{E}_1$ without loss of generality.

we can choose the x-axis lie along $\vec{k}$ and $\vec{E}$ and z-axis to lie along $\vec{B}_0$. Thus we have

$$k_y = k_z = E_y = E_z = 0$$

and

$$\hat{k} = k\hat{x}, \quad \vec{E}_1 = E_1\hat{x}$$



now we write eq^n (4) to (6) componentwise we get

$$\frac{\partial n_{e1}}{\partial t} + n_0 \frac{\partial v_{e1x}}{\partial x} = 0 \quad \text{—— (7)}$$

$$\frac{\partial v_{e1x}}{\partial t} = -\frac{e}{m_e}E_{1x} - \frac{e}{m_e c}v_{e1y}B_0 \quad \text{—— (8)}$$

$$\frac{\partial v_{e1y}}{\partial t} = \frac{e}{m_e c}v_{e1x}B_0 \quad \text{—— (9)}$$

$$\frac{\partial v_{e1z}}{\partial t} = 0 \quad \text{—— (10)}$$

$$\frac{\partial E_{1x}}{\partial x} = -4\pi e\, n_{e1} \quad \text{—— (11)}$$